Integrated industry-level and aggregate TFP-measures: Different approaches

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Abstract

There are two similar but in fact different methods, characterised in the present paper as the deliveries to final demand approach and the value-added approach, for measuring the economy level rate of TFP-growth and aggregating the KLEMS type of industry level measures to the economy level. Both industry level and economy level measures as well as the aggregation rules for both approaches, in the case of an open economy with nonzero net taxes on products in intermediate uses, are derived starting from the accounting identity for an industry. The consequences of different industries facing different prices are demonstrated. Empirical counterparts based on Laspeyres indices on the one hand and on the Törnqvist indices on the other, of the theoretical system are derived and compared with it. Other relevant choices in the empirical application are discussed and finally some preliminary results of the calculations based on Finnish data presented.

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1. Introduction

Measurement of total factor productivity growth, in its present form, has its roots, mainly, in Solow's (1957) seminal article, in which he demonstrated the equivalence of the economy level TFP-growth and the shift of the production function starting from theory of production and producer behaviour. Jorgenson and Griliches (1967) on the other hand started from the accounting identity equalising the total value of outputs with the total value of inputs. They aimed at homogeneous inputs and outputs and increased both the number of outputs and the number of inputs to obtain this. Their measure was, in principle, applicable to any production unit. They gave it an economic interpretation as the shift of production function by introducing the constant returns to scale production function and adding the necessary conditions of producer equilibrium in perfectly functioning markets.

In order to study the contribution of different industries to the economy level productivity growth it is necessary to have an aggregation rule showing in which way the economy level rate can be obtained from the industry level rates or alternatively decomposed into the industry level. Domar (1961) derived the aggregation rule in which each industry-level rate of TFP growth is "weighted by the ratio of the output of its industry to the value of the final product of the sector." Hulten (1978) proved the Domar aggregation rule in the case of a closed economy, in which prices paid by the users are equal to those received by the producers and all industries pay identical prices for their primary inputs. Jorgenson, Gollop and Fraumeni (1987) in their seminal contribution to productivity measurement developed an aggregation rule in which neither of these assumptions are needed. But their system requires assumptions about the existence of at least industry level value added functions. Their aggregation rule was similar to, but not identical with, the Domar aggregation, as will be shown in this paper.

Frank Gollop (1987) performed a systematic study of the two different approaches to the aggregation of industry level productivity measure to the economy level and, in fact, to the very definition of the economy level measure. He derived the aggregation/decomposition rules for TFP measures in an economy maximising the aggregate value of the deliveries to final demand on the one hand and for an economy maximising aggregate value added on the other, with the respective production possibilities frontiers as starting points. Unlike Hulten (1978), Gollop (1987) did not assume either closed economy or the equality between the prices received and paid for products used as intermediate inputs. Also Aulin-Ahmavaara

(2003) discussed the need to take into account the product taxes and subsidies on intermediate inputs in productivity measurement based on national accounts.

In his paper Gollop (1987) did however assume that 1) at the economy level product taxes less subsidies on intermediate inputs cancel out and 2) all the industries pay identical prices for a product used as intermediate input.[1] In the present paper also both of these assumptions are relaxed. Unlike Gollop (1987) we do not start from the production possibilities frontier and market equilibrium conditions. We are rather following the lead of Jorgenson and Griliches (1967) and start with the accounting identities.[2] The economy level and industry level TFP-measures as well as the aggregation rules corresponding to the two models introduced by Gollop are derived in section 2. Our theoretical system is based on the continuous time Divisia indices. In section 3 we discuss the choices that have to be made in the empirical application of this system. We derive two different systems for empirical application, one based on the, additive, Laspeyres indices and the other one based on the Törnqvist indices. We also report the preliminary results from the calculations based on the Finnish input output tables from years 2000 and 2001.

2. Productivity accounting: Two different approaches

2.1 Deriving the rate of TFP-growth from the accounting identity

The derivation of TFP or MFP measures can, following Jorgenson and Griliches (1967) start from the following accounting identity:

(1) $$\mathbf{q}'\mathbf{z} = \mathbf{p}'\mathbf{v}$$

where $\mathbf{q}$ is the price vector of outputs

$\mathbf{z}$ is the vector of output quantities

$\mathbf{p}$ is the price vector of inputs and

$\mathbf{v}$ is the vector of input quantities.

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[1] This is obvious for instance from his equation (19).

[2] The results derived from the accounting identities are compared with those obtained by Gollop (1987) in Aulin-Ahmavaara (2004).

The rate of total factor productivity growth is defined as the difference between the growth rates of outputs and inputs:

(2) $$d\log t = \sum_i \alpha_i d\log z_i - \sum_j \beta_j d\log v_j = \sum_j \beta_j d\log q_i - \sum_i \alpha_i d\log p_i ,$$

where α_i is the share of the i th output in total revenue and β_j the share of the j th input in total cost and $d\log y$ is the logarithmic time derivative of the variable y. This measure can be given an economic interpretation as the shift of the production function when a production function with constant returns to scale is assumed and all the relevant assumptions concerning markets and producer behaviour are made.

Following the SNA93 (ISWGNA, 1993) the accounting identity for an industry with only one type of output is defined as follows:

(3) $$\begin{aligned} q_j Q_j &= \sum_i q_i M_{ij} + (\sum_i p_{ij} M_{ij} - \sum_i q_i M_{ij}) \\ &+ \sum_i q_i^M M_{ij}^M + (\sum_i p_{ij}^M M_{ij}^M - \sum_i q_i^M M_{ij}^M) + \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj} \end{aligned},$$

Here the variables are

Q_j quantity of the output of the j th industry

q_j basic price of the output of the j th industry

M_{ij} quantity of the output of the i th industry used as intermediate input by the j th industry

p_{ij} purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the output of the i th industry it uses as intermediate input

M_{ij}^M quantity of the i th imported product used as intermediate input by the j th industry

q_i^M c.i.f. price of i th imported product

p_{ij}^M purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the i th imported product it uses as intermediate input

M_{ij}^M quantity of the i th imported product used by the j th industry as intermediate input

p_{kj}^K price paid by the j th industry for the capital input of category k

K_{kj} quantity of the capital input of category k used by the j th industry

p^L_{lj} price paid by the j th industry for the labour input of category l

L_{lj} quantity of the labour input of category l used by the j th industry.

When trade and transport margins are treated as separate inputs then the only difference between basic prices and purchasers' prices are taxes and subsidies on products.

Applying the formula in equation (2) to the accounting identity in equation (3) gives the rate of industry level TFP change:

(4)
$$\begin{aligned} d\log t_j = (q_j Q_j)^{-1}[q_j Q_j d\log Q_j - \sum_i q_i M_{ij} d\log M_{ij} - \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij} \\ - \sum_i q_i^M M^M_{ij} d\log M^M_{ij} - \sum_i (p^M_{ij} - q^M_i) M^M_{ij} d\log M^M_{ij} - \sum_k p^K_{kj} K_{kj} d\log K_{kj} - \sum_l p^L_{lj} L_{lj} d\log L_{lj})] \end{aligned}.$$

2.2 Deliveries to final demand approach

Deliveries to final demand consist of different products valued using some specified price concept. The options are basic prices, producers' prices and purchasers' prices (for the definitions, see ISWGNA, 1993). Here we have chosen basic prices since they represent the prices received by the producers. In order to calculate the value of the deliveries to final output using a specified price concept it is necessary to value also the output as well as the interindustry deliveries using the same price concept.[3] The accounting identity for an industry/ product in this case is

(5)
$$q_j Y_j = q_j Q_j - \sum_i q_j M_{ji} .$$

If we wish to consider the economy as one unit of production, then we obviously have to assume that all the industries face identical prices for their inputs. The quantity Z and the price p^Z of an input at the economy level can, in line with JGF (1987), then be defined, on the basis of the industry level quantities and prices, as follows:

[3] For more on this, see Aulin-Ahmavaara 2003.

(6) $$\sum_j p_j^Z Z_j = p^Z \sum_j Z_j = p^Z Z .$$

Summing over industries in equation (5) and (3) and substituting the former sum into the latter one results, in view of equation (6), in the following economy level accounting equations:

(7) $$\begin{aligned}\sum_j q_j Y_j &= \sum_j q_j Q_j - \sum_j \sum_i q_j M_{ji} \\ &= \sum_i (p_i - q_i) M_i + \sum_i q_i^M M^M{}_i + \sum_i (p_i^M - q_i^M) M_i^M + \sum_k p_k^K K_k + \sum_l p_l^L L_l .\end{aligned}$$

The economy level rate of productivity change is now obtained, from equation (7), by applying the formula in equation (2):

(8) $$\begin{aligned}d\log T = (\sum_j q_j Y_j)^{-1}[\sum_j q_j Y_j d\log Y_j - \sum_i (p_i - q_i) M_i d\log M_i - \sum_i q_i^M M^M{}_i d\log M_i - \\ \sum_i (p^M{}_i - q_i^M) M_i^M d\log M_i^M - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l].\end{aligned}$$

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The second term in the square brackets disappears regardless of the rates of growth of individual intermediate inputs if $p_i = q_i$ for all values of i i.e. if there are no taxes or subsidies on products in intermediate uses. On the other hand if $p_i \neq q_i$ for some domestic intermediate inputs the value of the term depends on the rates of growth of individual intermediate inputs. Likewise the fourth term in the brackets disappears if there are no product taxes (e.g. import duties) or subsidies on imported intermediate inputs. We have now obtained:

Result 1. *When the output of an economy is represented by the deliveries to final demand valued at basic prices the economy level rate of TFP growth depends, besides the rates of growth of these deliveries as well as the growth rates of labour and capital inputs, also on the rates of growth of imported intermediate inputs. Unless taxes and subsidies on intermediate inputs are non-existent, it depends also on the growth rates of individual domestic intermediate inputs.*

To establish the relation between the industry level measures and the economy level measure we multiply each industry level rate of TFP growth in equation (4) by the value of the industry's output and sum over industries to obtain:

(9)
$$\begin{aligned}\sum_j q_j Q_j d\log t_j &= \sum_j q_j Q_j d\log Q_j - \sum_j \sum_i q_i M_{ij} d\log M_{ij} - \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij} \\ &- \sum_j \sum_i q_i^M M_{ij}^M d\log M_{ij}^M - \sum_j \sum_i (p_{ij}^M - q_i^M) M_{ij}^M d\log M_{ij}^M \\ &- \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{lj}.\end{aligned}$$

Dividing both sides of (9) by the aggregate value of the deliveries to final demand, $\sum q_j Y_j$, and deducting resulting expression from both sides of equation (8) gives:

(10)
$$\begin{aligned} d\log T = (\sum_j q_j Y_j)^{-1} [\sum_j q_j Q_j d\log t_j \\ &- (\sum_i (p_i - q_i) M_i d\log M_i - \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij}) \\ &- (\sum_i (p_i^M - q_i^M) M_i^M d\log M_i^M - \sum_j \sum_i (p_{ij}^M - q_i^M) M_{ij}^M d\log M_{ij}^M) \\ &- (\sum_k p_k^K K_k d\log K_k - \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj}) \\ &- (\sum_l p_l^L L_l d\log L_l - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{lj})]\end{aligned}$$

The contribution of the industry level rates of TFP growth is represented by the first term in square brackets. The rest of the terms represent the contribution of the reallocation of the inputs by industry. If the price of an input Z is identical for all the industries, i.e. if $p_j^Z = p^Z$ for all values of j, then it follows directly from the definition in equation (6) that:

(11)
$$\sum_j p_j^Z Z_j d\log Z_j = \sum_j p^Z dZ_j = p^Z dZ = p^Z Z \, d\log Z .$$

Substituting this result into equation (10) shows that in this case the overall rate of TFP growth does not depend on the reallocation of the input by industry. This leads to our:

Result 2. *In the deliveries to final demand approach the rate of economy level TFP growth consists of 1) the weighted sum of the industry-level rates of TFP-growth with the ratios of the industries' outputs to the total value of deliveries to final demand as weights and 2) terms that reflect reallocation of capital, labour and intermediate inputs, both domestic and imported, by industry. However, if all the industries pay identical price for an individual input, the rate of the economy level TFP-growth does not depend on the reallocation of that input by industry*

Whether or not all the industries pay identical prices for their capital and labour inputs is, more or less, an empirical question. With perfect markets one could expect this to be the case, but markets hardly are perfect enough to produce exactly identical prices. Besides the classification of these inputs to different categories is not likely to be dense enough to produce identical prices even with perfect markets. This means that part of the differences in the distribution of these inputs by type of the input, e.g. by type of labour, actually appear as price differences. As to differences in the prices of intermediate inputs caused by taxes and subsidies on products, they often, but not always, can be expressed as a percentage of the value of the input. If this were the case there would be no price differences in intermediate inputs caused by taxes or subsidies on products, if all industries were facing the same taxes and subsidies. However, for instance in countries with VAT-system, some of the industries may have to pay VAT on their inputs, while others are exempted, depending whether or not their outputs are liable to VAT. Besides, the classification of intermediate inputs is not likely to be dense enough to make the categories homogenous with respect to possible rates of taxes/ subsidies.

2.3 Value added approach

Value added at the industry level equals the value of industry output valued at basic prices less the value of intermediate inputs valued at purchasers' prices:[4]

$$(12) \qquad v_j V_j = q_j Q_j - \sum_i p_{ij} M_{ij} - \sum_i M_{ij}^M = \sum_k p_{kj}^K K_{kj} + \sum_l p_{lj}^L L_{lj} \ ,$$

[4] In fact value added also includes other (than product) taxes less subsidies on production, that have to be allocated to the capital and labour inputs. We are here assuming that the entire operating surplus, (added by taxes on production relating to capital input) can be interpreted as capital compensation.

Summing over industries and assuming that all the industries pay identical prices for their capital and labour inputs gives:

(13) $$\sum_j v_j V_j = \sum_j q_j Q_j - \sum_j \sum_i p_{ij} M_{ij} - \sum_j \sum_i p_{ij}^M M_{ij}^M = \sum_k p_k^K K_k + \sum_l p_l^L L_l .$$

Substituting this in equations (7) produces

(14) $$\sum_j q_j Y_j = \sum_j v_j V_j + \sum_i (p_i - q_i) M_i + \sum_i q_i^M M_i^M + \sum_i (p_i^M - q_i^M) M_i^M \, .$$

Accordingly, the sum of industry value added and the sum of the values of the deliveries to final demand are equal if and only if there are no imported intermediate inputs and the aggregate value of taxes and subsidies on products in intermediate uses equals zero.

Applying the definition of the rate of TFP-change in equation (2) to the industry level accounting identity in (12) and to the economy level accounting identity in (13) produces the industry level rate of TFP-growth

(15) $$d\log t_j^v = (v_j V_j)^{-1} [v_j V_j d\log V_j - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{lj}] ,$$

and the aggregate rate of TFP growth based on the value-added approach

(16) $$d\log T^v = (\sum_j v_j V_j)^{-1} [(\sum_j v_j V_j d\log V_j - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l)] \cdot$$

Multiplying both sides of (15) by the ratio $(v_j V_j)(\sum_j v_j V_j)^{-1}$, summing over industries and subtracting the result from both sides of equation (16) produces

This is interpretation is problematic. For on more on this, see e.g. Diewert (2003) and Aulin-Ahmavaara (2003).

$$(17)\qquad \begin{aligned} d\log T^{v} &= (\sum_j v_j V_j)^{-1}[\sum_j v_j V_j d\log t_j^{v} \\ &- (\sum_k p_k^K K_k d\log K_k - \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj}). \\ &- (\sum_l p_l^L L_l d\log L_l - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{ij})] \end{aligned}$$

This equation gives the economy level measure based on the value added approach in terms of the industry level measures based on the value added.

Taking the logarithmic derivative of the expression in equation (12) produces:

$$(18)\qquad v_j V_j d\log V_j = q_j Q_j d\log Q_j - \sum_i p_{ij} M_{ij} d\log M_{ij} - \sum_i p_{ij}^M M_{ij}^M d\log M_{ij}^M \ .$$

Substituting this in equation (15) gives in view of equation (4) the following relation between the two industry level measures:

$$(19)\qquad d\log t_j^{v} = (v_j V_j)^{-1}(q_j Q_j) d\log t_j \ .$$

Substituting (19) into (17) produces an expression for the relationship between the aggregate value-added based rate of TFP-growth and the respective industry level rates, expressed in terms of total output:

$$(20)\qquad \begin{aligned} d\log T^{v} &= (\sum_j v_j V_j)^{-1}[\sum_j q_j Q_j d\log t_j \\ &- (\sum_k p_k^K K_k d\log K_k - \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj}). \\ &- (\sum_l p_l^L L_l d\log L_l - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{ij})] \end{aligned}$$

This together with equation (11) provides our

Result 3. *In the value added approach the rate of economy level TFP growth consists of 1) the weighted sum of the industry level rates of TFP growth with the rates of industries' outputs to the aggregate value added as weights and 2) terms that reflect the reallocation of* *capital and labour inputs by industry. However, if all the industries pay identical price for an individual input, the rate of the economy level TFP-growth does not depend on the reallocation of that input by industry. Taxes and subsidies on products do not appear in equation (17) and therefore the rate of the aggregate TFP growth does not, in this case, depend on the reallocation of intermediate inputs by industries.*

Multiplying this by the ratio $\sum v_j V_j / \sum q_j Y_j$ and substituting the result into (10) gives an expression for the rate of aggregate TFP-growth based on the deliveries to final demand approach in terms of the rate of aggregate TFP-growth based on the value added approach:

$$
\begin{aligned}
(21)\qquad & d\log T = (\sum_j q_j Y_j)^{-1}[(\sum_j v_j V_j) d\log T^v \\
& -(\sum_i (p_i - q_i) M_i d\log M_i - \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij}) \\
& -(\sum_i (p^M_i - q^M_i) M^M_i d\log M^M_i - \sum_j \sum_i (p^M_{ij} - q^M_i) M^M_{ij} d\log M^M_{ij})]
\end{aligned}
$$

The second line and third line disappear if industries pay identical prices for their domestically produced intermediate inputs as well as for their imported intermediate inputs. This can be concluded from equation (11). On the hand it is obvious from equation (14) that

$$
(22)\qquad \sum_j q_j Y_j = \sum_j v_j V_j \ \text{ iff } \sum_i (p_i - q_i) M_i + \sum_j \sum_i q^M_i M^M_{ij} + \sum_i (p^M_i - q^M_i) M^M_i = 0 \,.
$$

From (21) and (22) we obtain:

Result 4. *If all the industries pay identical prices for their intermediate inputs, both domestic and imported, the difference between the two economy level rates of growth depends only on the ratio of value of deliveries to final demand to value of total value added. This ratio depends on the aggregate value of imported intermediate inputs and of the aggregate value of product taxes less subsidies on intermediate inputs. If the prices paid by different industries for intermediate inputs are not identical, then the difference between the two economy level rates of TFP growth depends also on the reallocation of these inputs by industry.*

Until now we have only been dealing with the sum of the industries' value added. Next we shall define the price and the quantity of the economy level value added by the following expression:

(23) $$vV = v\sum_j V_j = \sum_j v_j V_j$$

Then the economy level accounting identity can be written as follows:

(24) $$vV = \sum_k p_k^K K_k + \sum_l p_l^L L_l \, .$$

Again applying the formula of equation (2) to this expression gives the economy level rate of TFP-growth:

(25) $$d\log T^W = (vV)^{-1}(vVd\log V - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l)$$

The relationship between the industry level and the economy level measures is obtained following the familiar procedure by forming the weighted average of the industry level value added based measures in equation (15), substituting equation (19) into the result and subtracting it from both sides of equation (25):

(26) $$\begin{aligned} d\log T^W &= (vV)^{-1}[\sum_j q_j Q_j d\log t_j \\ &- (vVd\log V - \sum_j v_j V_j d\log V_j) \\ &- (\sum_k p_k^K K_k d\log K_k - \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj}) \\ &- (\sum_l p_l^L L_l d\log L_l - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{lj})] \end{aligned} \, .$$

The second term in the square brackets now represents the effects of reallocation of value added. On the basis of equation (11) it is again obvious that the reallocation terms disappear if the prices of value added are identical for all of the industries or if the rates of growth of value added are identical in all of the industries. In the latter case, of course, no reallocation takes place. Also the two last rows disappear if all the industries pay the same prices for their capital and labour inputs or if the rates of growth of the quantities of these inputs are identical in all the industries, i.e. if no reallocation takes place. Thus equations (26) and (11) together provide us with:

Result 5. *If the economy level value added is used as the output variable in the value added approach the economy level rate of TFP consists of 1) the weighted sum of the industry level rates of TFP growth with the ratios of industries' outputs to the aggregate value added as weights and 2) terms representing reallocation of the value added as well as of capital and labour inputs by industry. If prices of the value added or of an individual input are identical for all industries then the economy rate of TFP growth does not depend on the reallocation of value added/that of the input.*

3. Empirical application

For an empirical application the theoretical continuous-time formulas based on Divisia-indexes have to be replaced by a discrete approximation. The natural candidate for the discrete approximation is the Törnqvist or translog productivity index and respective quantity indexes for inputs and outputs used by Christensen and Jorgenson (1970) as well as by JGF (1987). Diewert (1976) has shown Törnqvist index to be exact for translog aggregator function and Caves et al (1982) have presented strong economic support for the use Törnqvist productivity index. From our point of view it however has the shortcoming that it is not, unlike the theoretical Divisia index, additive. Therefore we developed an alternative empirical system, and performed the respective calculations, based on the Laspeyres indexes, which were also used by Jorgenson and Griliches (1967) and later also e.g. by Stenbæk and Sørensen (2004) in their study on the productivity development in Denmark.

3.1 Based on Laspeyres indexes

The Laspeyres quantity indices of industry level outputs and inputs are defined as follows:

Output $$Q_j = \frac{q_j^0 Q_j^1}{q_j^0 Q_j^0},$$

Domestic intermediate input at basic prices $$M_j = \frac{\sum_i q_i^0 M_{ij}^1}{\sum_i q_i^0 M_{ij}^0}$$

Net product taxes on domestic intermediate inputs $$S_j = \frac{\sum_i s_{ij}^0 q_i^0 M_{ij}^1}{\sum_i s_{ij}^0 q_i^0 M_{ij}^0}$$

Imported intermediate inputs $$M_j^M = \frac{\sum_i q_i^{M0} M_{ij}^{M1}}{\sum_i q_i^{M0} M_{ij}^{M0}}$$

Net product taxes on imported intermediate inputs $$S_j^M = \frac{\sum_i s_{ij}^{M0} q_i^0 M_{ij}^{M1}}{\sum_i s_{ij}^{M0} q_i^0 M_{ij}^{M0}}$$

Labour input $$L_j = \frac{\sum_l p_{lj}^0 L_{lj}^1}{\sum_l p_{lj}^0 L_{lj}^0}$$

Capital input $$K_j = \frac{\sum_k p_{kj}^0 K_{kj}^1}{\sum_k p_{kj}^0 K_{kj}^0}.$$

The rate industry level TFP growth is then defined, in the usual way, as the difference of the growth rate of the output and that of the weighted average of the inputs, with the value shares of the inputs as weights. After some manipulation we get:

$$(27)\quad \begin{aligned}\Delta t_j &= \frac{q_j^0 Q_{0j}^1 - q_j^0 Q_{0j}^0}{q_j^0 Q_j^0} - \frac{\sum_i (q_i^0 M_{ij}^1 - q_i^0 M_{ij}^0)}{q_j^0 Q_j^0} - \frac{\sum_i (s_{ij}^0 q_i^0 M_{ij}^1 - s_{ij}^0 q_i^0 M_{ij}^0)}{q_j^0 Q_j^0} \\ &- \frac{\sum_i (q_i^{M0} M_{ij}^{M1} - q_i^M M_{ij}^{M0})}{q_j^0 Q_j^0} - \frac{\sum_i (s_{ij}^{M0} q_i^{M0} M_{ij}^{M1} - s_{ij}^{M0} q_i^M M_{ij}^{M0})}{q_j^0 Q_j^0} \\ &- \frac{\sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{q_j^0 Q_j^0} - \frac{\sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{q_j^0 Q_j^0}\end{aligned},$$

which is a discrete counterpart of the theoretical industry level rate of TFP growth in equation (4).

The Laspeyres quantity indices at the aggregate level for the deliveries to final demand approach are defined as follows:

Gross output at basic prices $$Q = \frac{\sum_i q_i^0 Q_i^1}{\sum_i q_i^0 Q_i^0}$$

Intermediate deliveries of domestic products $$M = \frac{\sum_i q_i^0 M_i^1}{\sum_i q_i^0 M_i^0}$$

Deliveries to final demand of domestic products $$Y = \frac{\sum_i q_i^0 Y_i^1}{\sum_i q_i^0 Y_i^0}$$

Net product taxes on domestic deliveries to intermediate uses $$S = \frac{\sum_i s_i^0 q_i^0 M_i^1}{\sum_i s_i^0 q_i^0 M_i^0}$$

Imported intermediate inputs $$M^M = \frac{\sum_i q_i^{M0} M_i^{M1}}{\sum_i q_i^{M0} M_i^{M0}}$$

Net product taxes on imported intermediate input $$S^M = \frac{\sum_i s_i^{M0} q_i^0 M_i^{M1}}{\sum_i s_i^{M0} q_i^0 M_i^{M0}}$$

Labour input $$L = \frac{\sum_l p_l^0 L_l^1}{\sum_l p_l^0 L_l^0}$$

Capital input $$K = \frac{\sum_k p_k^0 K_k^1}{\sum_k p_k^0 K_k^0}.$$

As can be easily verified the percentage change in deliveries to final demand is obtained as follows:

$$(28) \qquad \Delta Y = \frac{\sum_i q_i^0 Q_i^1 - \sum_i q_i^0 Q_i^0}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i q_i^0 M_i^1 - \sum_i q_i^0 M_i^0}{\sum_i q_i^0 Y_i^0} = \frac{\sum_i q_i^0 Y_i^1 - \sum_i q_i^0 Y_i^0}{\sum_i q_i^0 Y_i^0}.$$

The rate of the economy level TFP growth is then defined as the difference of the growth rate of the deliveries to final demand and the weighted average of the growth rates of the aggregate inputs, with the value shares of the inputs as weights. After some simplification this gives:

$$(29) \qquad \begin{aligned} \Delta T = {} & \frac{\sum_i q_i^0 Y_i^1 - \sum_i q_i^0 Y_i^0}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (s_i^0 q_i^0 M_i^1 - s_i^0 q_i^0 M_i^0)}{\sum_i q_i^0 Y_i^0} \\ & - \frac{\sum_i (q_i^{M0} M_i^{M1} - q_i^M M_i^{M0})}{\sum_i q_i^0 Y_i^0} - \frac{\sum_i (s_i^{M0} q_i^{M0} M_i^{M1} - s_i^{M0} q_i^M M_i^{M0})}{\sum_i q_i^0 Y_i^0}, \\ & - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_k (p_k^0 K_{kj}^1 - p_k^0 K_k^0)}{\sum_i q_i^0 Y_i^0} \end{aligned}$$

which is a discrete version of the theoretical economy level rate of TFP growth in equation (8).

Domar-aggregation can be performed in the familiar way by multiplying the industry level rates in equation (27) by the ratio of industry gross output to the economy level deliveries to final demand, summing over industries and subtracting the result from both sides of equation (29). As can be easily verified

$$\frac{\sum_i (q_i^{M0} M_i^{M1} - q_i^{M} M_i^{M0})}{\sum_i q_i^0 Y_i^0} = \frac{\sum_j \sum_i (q_i^{M0} M_{ij}^{M1} - q_i^{M} M_{ij}^{M0})}{\sum_i q_i^0 Y_i^0} \tag{30}$$

and

$$\sum_j \frac{q_j^0 Q_j^0}{\sum_i q_i^0 Y_i^0} \left(\frac{q_j^0 Q_{0j}^1 - q_j^0 Q_{0j}^0}{q_j^0 Q_j^0} - \frac{\sum_i (q_i^0 M_{ij}^1 - q_i^0 M_{ij}^0)}{q_j^0 Q_j^0} \right) = \frac{\sum_i q_i^0 Y_i^1 - \sum_i q_i^0 Y_i^0}{\sum_i q_i^0 Y_i^0} . \tag{31}$$

We then have the following decomposition of the economy level rate of TFP growth to the contributions of industry level rates and of the reallocation of labour and capital inputs as well as of the reallocation of net taxes on products in intermediate uses.

$$\begin{aligned}
\Delta T = {} & \sum_j \frac{q_j^0 Q_j^0}{\sum_i q_i^0 Y_i^0} \Delta t_j \\
& - \left(\frac{\sum_i (s_i^0 q_i^0 M_i^1 - s_i^0 q_i^0 M_i^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_j \sum_i (s_{ij}^0 q_i^0 M_{ij}^1 - s_{ij}^0 q_i^0 M_{ij}^0)}{\sum_i q_i^0 Y_i^0} \right) \\
& - \left(\frac{\sum_i (s_i^{M0} q_i^{M0} M_i^{M1} - s_i^{M0} q_i^{M} M_i^{M0})}{\sum_i q_i^0 Y_i^0} - \frac{\sum_j \sum_i (s_{ij}^{M0} q_i^{M0} M_{ij}^{M1} - s_{ij}^{M0} q_i^{M} M_{ij}^{M0})}{\sum_i q_i^0 Y_i^0} \right) \\
& - \left(\frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_j \sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{\sum_i q_i^0 Y_i^0} \right) \\
& - (\frac{\sum_k (p_k^0 K_k^1 - p_k^0 K_k^0)}{\sum_i q_i^0 Y_i^0} - \frac{\sum_j \sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{\sum_i q_i^0 Y_i^0}
\end{aligned} \tag{32}$$

Equation (32) is a discrete counterpart of equation (10). The second and third rows of its right hand side disappear if the tax rates are identical in all intermediate uses of domestic products as well in all intermediate uses of imported products. Equation (32) thus is in accordance with our result 2.

In the value added approach the growth rate of the industry level TFP is then defined as the difference of the growth rate of the industry's value added and that of the weighted average of the growth rates of aggregate labour and capital inputs, with the value shares of the inputs as weights.

$$(33) \qquad \Delta t_j^v = \frac{v_j^0 V_j^1 - v_j^0 V_j^0}{v_j^0 V_j^0} - \frac{\sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{q_j^0 V_j^0} - \frac{\sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{q_j^0 V_j^0},$$

which is a discrete counterpart of the theoretical equation (15). Since

$$(34) \qquad \begin{aligned} \Delta V_j &= \frac{v_j^0 V_j^1 - v_j^0 V_j^0}{v_j^0 V_j^0} \\ &= \frac{q_j^0 Q_j^1 - \sum_i q_i^0 M_{ij}^1 - \sum_i s_{ij}^0 q_i^0 M_{ij}^1 - \sum_i q_i^{M0} M_{ij}^{M1} - \sum_i s_{ij}^{M0} q_i^{M0} M_{ij}^{M1}}{v_j^0 V_j^0} \\ &- \frac{q_j^0 Q_j^0 - \sum_i q_i^0 M_{ij}^0 - \sum_i s_{ij}^0 q_i^0 M_{ij}^0 - \sum_i q_i^{M0} M_{ij}^{M0} - \sum_i s_{ij}^{M0} q_i^{M0} M_{ij}^{M0}}{v_j^0 V_j^0} \end{aligned}$$

we have

$$(35) \qquad \Delta t_j^v == \frac{q_j^0 Q_j^0}{v_j^0 V_j^0} \Delta t_j,$$

which again is a discrete counterpart of equation (19).

Next the rate of economy level TFP growth is defined as follows:

$$(36) \qquad \Delta T^{v} = \frac{\sum_j v_j^0 V_j^1 - \sum_j v_j^0 V_j^0}{\sum_j v_j^0 V_j^0} - \frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_k (p_k^0 K_{kj}^1 - p_k^0 K_k^0)}{\sum_j v_j^0 V_j^0}$$

which again is in accordance with the theoretical rate of growth in equation (16).

Multiplying both sides of equation (33) by the ratio of industry's value added to sum of the value added of all the industries and subtracting the result from both sides of equation (36) gives

$$(37) \qquad \begin{aligned} \Delta T^{v} &= \sum_j \frac{v_j^0 V_{ji}^0}{\sum_j v_j^0 V_j^0} \Delta t_j^v \\ &- \left(\frac{\sum_l (p_l^0 L_l^1 - p_l^0 L_l^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_j \sum_l (p_{lj}^0 L_{lj}^1 - p_{lj}^0 L_{lj}^0)}{\sum_j v_j^0 V_j^0}\right) \quad , \\ &- \left(\frac{\sum_k (p_k^0 K_k^1 - p_k^0 K_k^0)}{\sum_j v_j^0 V_j^0} - \frac{\sum_j \sum_k (p_{kj}^0 K_{kj}^1 - p_{kj}^0 K_{kj}^0)}{\sum_j v_j^0 V_j^0}\right. \end{aligned}$$

which is a discrete version of the theoretical equation (17) and confirms our result 3.

Thus we have been able to derive a system based on Laspeyres indices, which is in accordance with our theoretical formulas and confirms our theoretical results.

3.2 Based on Törnqvist indices

We define the following logarithms of industry-level Törnqvist quantity indices:

Output at basic prices $\Delta \log Q_j = \log q_j^0 Q_j^1 - \log q_j^0 Q_j^0$,

Deliveries to final demand at basic prices $\Delta \log Y_j = \log q_j^0 Y_j^1 - \log q_j^0 Y_j^0$,

Average value share $\overline{\frac{v_i}{\sum_i v_i}} = \left(\frac{v_i^0}{\sum_i v_i^0} + \frac{v_i^1}{\sum_i v_i^1}\right) 1/2$, Domestic intermediate inputs at basic prices $\Delta \log M_j = \sum_i (\overline{\frac{q_i M_{ij}}{\sum_i q_i M_{ij}}} \Delta \log M_{ij})$,

Net product taxes on domestic intermediate inputs $\Delta \log S_j = \sum_i (\overline{\frac{S_{ij}}{\sum_i S_{ij}}} \Delta \log M_{ij})$

Imported intermediate inputs $\Delta \log M_j^M = \sum_i (\overline{\frac{q_i^M M_{ij}^M}{\sum_i q_i^M M_{ij}^M}} \Delta \log M_{ij}^M)$

Net product taxes on imported intermediate inputs $\Delta \log S_j^M = \sum_i (\overline{\frac{S_{ij}^M}{\sum_i S_{ij}^M}} \Delta \log M_{ij}^M)$

Labour input $\Delta \log L_j = \sum_l (\overline{\frac{p_{lj} L_{lj}}{\sum_l p_{lj} L_{lj}}} \Delta \log L_{lj})$

Capital input $\Delta \log K_j = \sum_k (\overline{\frac{p_{kj} K_{kj}}{\sum_k p_{kj} K_{kj}}} \Delta \log K_{kj})$.

Rate of industry level TFP-change is then defined as follows:

(38)
$$\begin{aligned}\Delta \log t_j = {} & \Delta \log Q_j - (\overline{\frac{\sum_i q_i M_{ij}}{q_j Q_j}} \Delta \log M_j) - (\overline{\frac{\sum_i S_{ij}}{q_j Q_j}} \Delta \log S_j) \\ & - (\overline{\frac{\sum_i q_i^M M_{ij}^M}{q_j Q_j}} \Delta \log M_j^M) - (\overline{\frac{\sum_i S_{ij}^M}{q_j Q_j}} \Delta \log S_j^M) \\ & - (\overline{\frac{\sum_l p_{lj} L_{lj}}{q_j Q_j}} \Delta \log L_j) - (\overline{\frac{\sum_k p_{kj} K_{kj}}{q_j Q_j}} \Delta \log K_j) \quad .\end{aligned}$$

For the economy level we define the following logarithms of Törnqvist quantity indices:

Deliveries to final demand of domestic products $\Delta \log Y = \sum_i \overline{\frac{q_i Y_i}{\sum_i q_i Y_i}} \Delta \log Y_i$

Gross output at basic prices $\Delta \log Q = \sum_i \overline{\frac{q_i Q_i}{\sum_i q_i Q_i}} \Delta \log Q_i$

Deliveries to intermediate uses of domestic products $\Delta \log M = \sum_i \frac{\overline{q_i M_i}}{\sum_i q_i M_i} \Delta \log M_i$

Net product taxes on domestic intermediate inputs $\Delta \log S = \sum_i (\frac{\overline{S_i}}{\sum_i S_i} \Delta \log M_i)$

Imported intermediate inputs $\Delta \log M^M = \sum_i (\frac{\overline{q_i M_i^M}}{\sum_i q_i M_i^M} \Delta \log M_i^M)$

Net product taxes on imported intermediate inputs[5] $\Delta \log S^M = \sum_i (\frac{\overline{S_i^M}}{\sum_i S_i^M} \Delta \log M_i^M)$

Labour input $\Delta \log L = \sum_l (\frac{\overline{p_l L_l}}{\sum_l p_l L_l} \Delta \log L_l)$

Capital input $\Delta \log K = \sum_k (\frac{\overline{p_k K_k}}{\sum_k p_k K_k} \Delta \log K_k)$

The rate of economy level TFP growth is defined as

(38)
$$\begin{aligned}\Delta \log T = \Delta \log Y &- (\frac{\overline{\sum_i S_i}}{\sum_i q_i Y_i} \Delta \log S) - (\frac{\overline{\sum_i q_i^M M_i^M}}{\sum_i q_i Y_i} \Delta \log M^M) \\ &- (\frac{\overline{\sum_i S_i^M}}{\sum_i q_i Y_i} \Delta \log S^M) - (\frac{\overline{\sum_l p_l L_l}}{\sum_i q_i Y_i} \Delta \log L) - (\frac{\overline{\sum_k p_k K_k}}{\sum_i q_i Y_i} \Delta \log K)\end{aligned}.$$

For Domar-aggregation both sides of equation (38) for industry level TFP growth are multiplied by following aggregation coefficients:

(40)
$$C_j = \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \frac{\overline{q_j Q_j}}{\sum_j q_j Q_j}$$

---

[5] In fact taxes on products and subsidies on products were calculated as two separate terms.

By summing over industries and subtracting the result from both sides of equation (39) for the economy level we obtain:

$$
\begin{aligned}
&\Delta \log T = \sum_j (C_j \Delta \log t_j) \\
&+\Delta \log Y - (\sum_j (C_j \Delta \log Q_j) - \sum_j (C_j \overline{\frac{\sum_i q_i M_{ij}}{q_j Q_j}} \Delta \log M_j)) \\
&-(\overline{\frac{\sum_i S_i}{\sum_i q_i Y_i}} \Delta \log S - \sum_j (C_j \overline{\frac{\sum_i S_{ij}}{q_j Q_j}} \Delta \log S_j)) \\
&-(\overline{\frac{\sum_i q_i^M M_i^M}{\sum_i q_i Y_i}} \Delta \log M^M - \sum_j (C_j \overline{\frac{\sum_i q_i^M M_{ij}^M}{q_j Q_j}} \Delta \log M_j^M)) \\
&-(\overline{\frac{\sum_i S_i^M}{\sum_i q_i Y_i}} \Delta \log S^M - \sum_j (C_j \overline{\frac{\sum_i S_{ij}^M}{q_j Q_j}} \Delta \log S_j^M)) \\
&-(\overline{\frac{\sum_l p_l L_l}{\sum_i q_i Y_i}} \Delta \log L - \sum_j (C_j \overline{\frac{\sum_l p_{lj} L_{lj}}{q_j Q_j}} \Delta \log L_j)) \\
(41)\quad &-(\overline{\frac{\sum_k p_k K_k}{\sum_i q_i Y_i}} \Delta \log K - \sum_j (C_j \overline{\frac{\sum_k p_{kj} K_{kj}}{q_j Q_j}} \Delta \log K_j)) \quad .
\end{aligned}
$$

Unlike the respective theoretical equation (10) and equation (32) for Laspeyres indices equation (41) also includes a term (second row on the right hand side) that can be taken to represent the reallocation of industry output to final uses and intermediate uses in different industries. There is also another important difference between equations (10) and (32) on the one hand and equation (41) on the other. In equations (10) and (32) the reallocation terms disappear in the case of identical prices of the inputs in different uses. In the decomposition based on Törnqvist indexes this does not necessarily hold. This can be seen from the following example. Assume that is only one category of labour and that identical price (equal to 1) is paid for it in all uses and that the price does not change. Assume also that all the relevant value shares are very close to each other in the years of comparison. Then we have $\Delta \log L = \log \frac{\sum_j L_j^1}{\sum_j L_j^0}$, which should be equal to $\sum_j \overline{w}_j \log \frac{L_j^1}{L_j^0}$ for the reallocation term to disappear. Here w_j is the value share of the labour input of the j th industry in the total value of labour input. Since the direct sum is not a translog aggregator function there is no reason to expect reallocation term to disappear in this case[6], even if the prices paid for the labour input in different uses are identical. This means that there could be productivity gains from reallocation of labour to different industries even if the marginal rates of transformation, were identical. On the other hand one can of course, theoretically, assume that if prices were identical no reallocation would take place, but in practice this is not necessarily true.

From the definitions of growth rates of gross output and that of deliveries to intermediate demand we get an alternative expression for the growth rate of deliveries to final demand:

$$(42)\qquad \Delta\log Y_1 = \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i}\right)^{-1} \Delta\log Q - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i}\right)^{-1} \overline{\frac{\sum_j q_j M_j}{\sum_j q_j Q_j}} \Delta\log M\ .$$

Furthermore We get the following new expression for the second row of the right hand side of equation (41) by subtracting the left hand side of equation (42) from it and then adding the right hand side of equation (42):

$$(43)\qquad \begin{aligned} &\Delta\log Y - (\sum_j (C_j \Delta\log Q_j) - \sum_j (C_j \overline{\frac{\sum_i q_i M_{ij}}{q_j Q_j}} \Delta\log M_j)) \\ &= \Delta\log Y - \Delta\log Y_1 + \sum_j (C_j \overline{\frac{\sum_i q_i M_{ij}}{q_j Q_j}} \Delta\log M_j - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i}\right)^{-1} \overline{\frac{\sum_j q_j M_j}{\sum_j q_j Q_j}} \Delta\log M \end{aligned}$$

Thus the second row of the right hand side of the equation (41) in fact represents the difference between the two measures of final output added by the contribution of the reallocation of domestic intermediate inputs. The difference between the two measures is interpreted to reflect the reallocation of industry output between deliveries to final input and intermediate deliveries.

We also calculated an alternative rate of economy level TFP growth by replacing $\Delta\log Y$ by $\Delta\log Y_1$.

---

[6] The logarithm of the Törnqvist index is equal to the logarithm of the ratio of the value of the translog aggregator function in two subsequent periods. See e.g. Diewert (2000) p.7.

$$\Delta \log T_1 = \Delta \log Y_1 - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \left(\frac{\overline{\sum_j S_j}}{\sum_j q Q_j} \Delta \log S \right)$$

(44)

$$- \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \left(\frac{\overline{\sum_i q_i^M M_i^M}}{\sum_j q_j Q_j} \Delta \log M^M \right) - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \left(\frac{\overline{\sum_i S_i^M}}{\sum_j q_j Q_j} \Delta \log S^M \right)$$

$$- \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \left(\frac{\overline{\sum_l p_l L_l}}{\sum_j q_j Q_j} \Delta \log L \right) - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \left(\frac{\overline{\sum_k p_k K_k}}{\sum_j q_j Q_j} \Delta \log K \right)$$

Subtracting the weighted sum of the industry level measures from both sides of equation (44) gives a decomposition of the economy level measure. In this decomposition the second row on the right hand side is different from the one in equation (41), while the rest of the terms are identical in these two equations:

$$\Delta \log T_1 = \sum_j (C_j \Delta \log t_j)$$

(45)

$$+ \sum_j \left(C_j \frac{\overline{\sum_i q_i M_{ij}}}{q_j Q_j} \Delta \log M_j \right) - \left(\frac{\overline{\sum_i q_i Y_i}}{\sum_i q_i Q_i} \right)^{-1} \frac{\overline{\sum_j q_j M_j}}{\sum_j q_j Q_j} \Delta \log M \cdots$$

In this case, unlike in theory and in the case of Laspeyres indices, the reallocation terms include the reallocation of intermediate inputs at basic prices.

In the value added approach the rate of growth of the industry level value-added is defined as follows:

$$\Delta \log V_j = \left(\frac{\overline{v_j V_j}}{q_j Q_j} \right)^{-1} \Delta \log Q_j - \left(\frac{\overline{v_j V_j}}{q_j Q_j} \right)^{-1} \left(\frac{\overline{\sum_i q_i M_{ij}}}{q_j Q_j} \Delta \log M_j \right)$$

(46)

$$- \left(\frac{\overline{v_j V_j}}{q_j Q_j} \right)^{-1} \left(\frac{\overline{\sum_i S_{ij}}}{q_j Q_j} \Delta \log S_j \right) - \left(\frac{\overline{v_j V_j}}{q_j Q_j} \right)^{-1} \left(\frac{\overline{\sum_i q_i^M M_{ij}^M}}{q_j Q_j} \Delta \log M_j^M \right)$$

For the industry-level rate of TFP-change we thus get:

(47) $$\Delta\log t_j^v = \Delta\log V_j - \left(\frac{\overline{v_j V_j}}{q_j Q_j}\right)^{-1} \left(\frac{\overline{\sum_l p_{lj} L_{lj}}}{q_j Q_j} d\log L_j\right) - \left(\frac{\overline{v_j V_j}}{q_j Q_j}\right)^{-1} \left(\frac{\overline{\sum_k p_{kj} K_{kj}}}{q_j Q_j} d\log K_j\right)$$

This together with (38) and (46) yields the familiar relationship between the value added based and gross output based industry level measures:

(48) $$\Delta\log t_j^v = \left(\frac{\overline{v_j V_j}}{q_j Q_j}\right)^{-1} \Delta\log t_j .$$

The economy level rate of value added growth is defined as the weighted average of the industry level rates:

(49) $$\Delta\log V = \sum_j \overline{\frac{v_j V_j}{\sum_j v_j V_j}} \Delta\log V_j$$

This leads to the economy level rate of TFP growth defined as follows:

(50) $$\Delta\log T^v = \Delta\log V - \left(\overline{\frac{\sum_l p_l L_l}{\sum_j v_j V_j}} \Delta\log L\right) - \left(\overline{\frac{\sum_k p_k K_k}{\sum_j v_j V_j}} \Delta\log K\right) .$$

Multiplying both sides of the equation (47) by $\overline{\frac{v_j V_j}{\sum_j v_j V_j}}$ summing over industries and subtracting the result from both sides of (50) gives:

$$
\begin{aligned}
\Delta \log T^{v} &= \sum_{j} \Big(\overline{\frac{v_j V_j}{\sum_{j} v_j V_j}} \Delta \log t_j^{v} \Big) \\
&- \Big(\overline{\frac{\sum_{l} p_l L_l}{\sum_{j} v_j V_j}} \Delta \log L - \sum_{j} \Big(\overline{\frac{v_j V_j}{\sum_{j} v_j V_j}} \overline{\left(\frac{v_j V_j}{q_j Q_j} \right)}^{-1} \Big(\overline{\frac{\sum_{l} p_{lj} L_{lj}}{q_j Q_j}} \Delta \log L_j \Big)\Big) \\
&- \Big(\overline{\frac{\sum_{k} p_k K_k}{\sum_{j} q_j V_j}} \Delta \log K - \sum_{j} \Big(\overline{\frac{v_j V_j}{\sum_{j} v_j V_j}} \overline{\left(\frac{v_j V_j}{q_j Q_j} \right)}^{-1} \Big(\overline{\frac{\sum_{k} p_{kj} K_{kj}}{q_j Q_j}} \Delta \log K_j \Big)\Big)
\end{aligned}
\qquad (51) \quad ,
$$

which has the same terms as the respective theoretical equation (17) as well as the respective equation based on Laspeyres indices in (37). However, again, in the case of Törnqvist indices reallocation terms do not disappear even if different industries pay identical prices for their labour and capital inputs.

3.3 Calculations based on Finnish data

The theoretical results were tested by an empirical application with Finnish data for the years 2000 and 2001. There are several choices that have to be made in an empirical application. Each of them is likely to have an effect on the results of the calculations. The first one is whether to use supply and use tables or symmetric input-output tables. We were using SIOTs, mainly since it keeps the formulas somewhat more simple.

The second choice concerns the level of aggregation on which the calculations are performed as well as the level of aggregation on which the deflation is performed. E.g. the draft Eurostat Input-Output manual suggests the deflation to be performed at the lowest possible level. Our empirical exercise is based on the Finnish supply and use tables for 2000 and 2001, at current and fixed price, with about 950 different products and about 180 industries. Deflation was, for the fixed price tables, originally performed, and the tables balanced, at this detailed level. But if deflation is performed at a more detailed level than the actual calculations then the basic price at a more aggregate level (i.e. its rate of change) can be different in different uses, because these uses consist of different baskets of the more detailed level products. Therefore we redeflated the tables at the level of 55 industries used in the calculation of the productivity measures.

The third one is the choice of the price concept on which the deflator is based. E.g. in JGF (1987) outputs are valued at basic prices, to use the terminology of the SNA93, and inputs are valued at purchasers' prices without trade and transport margins, in other words at basic prices plus taxes, net of subsidies, on products. Each industry's output is deflated by its output deflator at basic price. The intermediate deliveries from an industry are deflated by an output deflator that includes the net taxes on products paid for that output. Including taxes net of subsidies on products in a deflator would require that the tax rate is the same in all uses of that product. In the case of the value added tax this is not normally true. Even in the case of the rest of the product taxes/subsidies there can be problems, since the product baskets in different uses consist of different detailed level products with, possibly, unequal tax rates. To be able to test our theoretical formulas we ended up deflating all the uses of the domestic output of an aggregated level product by its implicit output deflator at basic price. Respectively all the uses of an aggregated level imported product were deflated by its average deflator at basic (c.i.f.) price in all its uses. The growth rates of product taxes on intermediate inputs were assumed to be equal to the growth rates of respective inputs.

The fourth choice concerns the index number formula. We made the calculations both using the, additive Laspeyres indices and also using the, nonadditive but superlative, Törnqvist indices. The differences between the applications based on these two types of indices were discussed above.

Since the main purpose of our empirical exercise was to test our theoretical results concerning the different approaches of aggregating the industry level measures to the economy level, we did not make an effort to allocate our labour input to different categories. The labour compensation of the self-employed was estimated on the basis of the hourly compensation of the employees in each of the industries. The results did, as in fact could be expected, depend on the level of aggregation of the industries.

As to the capital input it is represented by the services of fixed capital and the indices were constructed, as far as possible, in line with JGF (1987). Productive capital stocks for different categories of fixed capital were constructed by the perpetual inventory method assuming geometric rates of efficiency decline. Net rates of return on capital were calculated ex post from operating surplus plus mixed income after subtracting the estimated labour compensation of the self-employed.

Table 1. Economy level TFP-growth and contributions to the economy level growth of output

	Deliveries to final demand approach			Value added approach	
	Laspeyres (29)*	Törnqvist 1 (39)*	Törnqvist 2 (44)*	Laspeyres (36)*	Törnqvist (50*)
Growth of output	0,954	1,036	0,975	1,609	1,528
Net taxes on domestic products in intermediate uses**	0,081	0,060	0,060		
Imported intermediate inputs	-0,322	-0,338	-0,338		
Net taxes on imported products in intermediate uses	0,008	0,004	0,004		
Labour	0,262	0,266	0,266	0,343	0,345
Capital	0,721	0,702	0,702	0,942	0,909
Economy level TFP growth	0,204	0,342	0,281	0,325	0,274

*Numbers in parentheses refer to the formulas on which the calculations in the column are based.

**In fact taxes and products and subsidies on products were calculated as two separate terms.

Table 2. Contributions to the economy level TFP growth

	Deliveries to final demand approach			Value added approach	
	Laspeyres (32)*	Törnqvist 1 (42)*	Törnqvist 2 (45)*	Laspeyres (37)*	Törnqvist (51)*
Industry level TFP growth	-0,270	-0,196	-0,196	-0,353	-0,263
Reallocation of					
- Industry output between final and intermediate uses		0,061			
- Intermediate uses of domestic products		0,116	0,116		
- Net taxes on domestic products in intermediate uses**	-0,037	-0,031	-0,031		
- Intermediate uses of imported products		-0,009	-0,009		
- Net taxes on imported products intermediate uses	-0,007	-0,012	-0,012		
- Labour	0,224	0,215	0,215	0,292	0,280
- Capital	0,295	0,198	0,198	0,386	0,259
Economy level rate of TFP growth	0,204	0,342	0,281	0,325	0,275

*Numbers in parentheses refer to the formulas on which the calculations in the column are based.

**In fact the contributions of taxes and subsidies on products were calculated as two separate terms.

The results of the calculations are given in Tables 1 and 2. The rate of output growth is higher in the value added approach (Table1) regardless of the index number formula used. The rates of economy level TFP growth, and accordingly the contributions of TFP growth the growth of output, do not actually differ very much in the two approaches. There are no contributions of net taxes on products or of imported intermediate inputs in the value added approach. The contributions of labour and capital inputs, again, are higher in the value added approach than in the deliveries to final demand approach.

The negative contribution of industry-level TFP growth to the economy level TFP growth is higher in the value added approach (Table 2). The calculations based on Törnqvist indexes include the contribution of the reallocation of in industries' output, at basic prices, between final and intermediate uses as well as the reallocation of intermediate uses of domestic inputs, at basic prices, between industries (Törnqvist 1) or alternatively only the latter (Törnqvist 2). Reallocation of outputs or intermediate uses at basic prices does not appear in the decomposition based on the Laspeyres indices. This is because we assumed identical basic prices in all uses. In the case of identical prices reallocation of inputs between industries does not contribute to the economy level productivity growth either theoretically or in the system based on Laspeyres indices. The contributions of reallocation of labour and capital inputs are smaller in the calculations based on Törnqvist indices than in those based on the Laspeyres indices. On the other hand the contributions of the reallocation of labour and capital inputs are higher in the value added approach than in the deliveries to final demand approach, regardless which of the index number formulas used in the calculations.

4. Concluding remarks

We have, in terms of the theoretical Divisia index and starting from the respective accounting identities, derived the industry-level as well as the economy level rates of total factor productivity growth based on the deliveries to final demand approach on the one hand on the value added approach on the other. It appeared that, in the case of the deliveries to final demand approach it was necessary, in addition to the terms representing capital and labour inputs, to include terms representing imported intermediate inputs as well as terms representing product taxes/subsidies on intermediate uses in the formula for economy level rate of TFP-growth.

We have also derived the aggregation rules from the industry-level to the economy level. The economy-level rates of TFP growth could, in both cases, be represented as weighted sums of the same industry-level rates added by reallocation terms. But the weights were different in different approaches. In the deliveries to final demand approach the weights were equal to the ratios of industries' outputs to the aggregate value of deliveries to final demand. In the value added approach they were equal to the ratios of industries' outputs to the aggregate value added. The terms representing reallocation of labour and capital inputs were needed, in the aggregation equation, in both cases. But in the deliveries to final demand approach also terms representing reallocation of, both imported and domestically produced, intermediate deliveries by industry, were required. In the end the difference in the aggregate TFP growth between these two approaches depends on the total value of imported inputs, the aggregate value of product taxes and subsidies on intermediate inputs, and on the reallocation of intermediate inputs by industry.

We also studied another variation of the value-added approach, in which the economy level output was represented by the economy level value added. In this case the aggregation equation required reallocation terms of value added as well as those of labour and capital inputs by industry.

The choice of the approach in the end, of course, depends on what we think the economy is maximising. If we are thinking of a national economy as a production unit it would seem natural to assume that it is maximising the value of the output gross of depreciation that it is able to deliver outside the unit with respect to the inputs it uses. But from the consumers' point of view the economy is assumed to be maximising its welfare. Weitzman (1976) has shown that consumption plus changes in net worth, i.e. final output net of depreciation, can be used as an indicator of the present value of future consumption. This issue is beyond the limits of the present paper. It is discussed e.g. by Hulten (1992 and 2001), who concludes that the appropriate welfare, i.e. NNP, based analysis is separate from and complementary to, the GDP based analysis of productive efficiency. Besides we also have to decide whether the value of the output is seen from the producers' point of view (basic prices) or from the consumers point of view (purchasers' prices). ten Raa and Mohnen (2002) are maximising the deliveries to domestic final demand gross of depreciation, of both domestic output and imported products. The proportions of the actual final demand are preserved. Also the observed proportions during any time period of course depend on the price concept used to measure the components of domestic final demand.

The theoretical results were tested by an empirical application to the Finnish data for the years 2000 and 2001. There are several choices that have to be made in an empirical application. Each of them is likely to have an effect on the results of the calculations. The first one is whether to use supply and use tables or symmetric input-output tables. We were using SIOTs, mainly since it keeps the formulas somewhat more simple .

The second one concerns the level of aggregation on which the calculations are performed as well as the level of aggregation on which the deflation is performed. If deflation is performed at a more detailed level than the actual calculations then the basic price at a more aggregate level (i.e. its rate of change) can be different in different uses, because these uses consist of different baskets of the more detailed level products. Also the choice of the price concept on which the deflator is based is of importance. Including taxes net of subsidies on products in a deflator would require identical tax rate in all uses of a product. In our case the same level of aggregation was used both in deflation and in calculations and deflation was performed at basic prices.

Finally the index number formula has to be decided. We derived the empirical application and made the calculations both using the, additive, Laspeyres indices and also using the, nonadditive, but superlative, Törnqvist indices. In the empirical system based on the Laspeyres indices we were able to exactly replicate the theoretical case and to get exactly the same results as in the theoretical case. In the system based on the Törnqvist indices results were different. This difference was caused by the fact that unlike in the theoretical case (Divisia indices) and in the case of Laspeyres indices, in the case of Törnqvist indices identical prices for an input in different industries does not mean that the reallocation terms would disappear. In the case of the Törnqvist indices they can only be expected to disappear if an input grows at identical rate in all its uses. In the case of the theoretical Divisia indices as well as in the case of the Laspeyres indices the reallocation of an input between industries does contribute to the economy-level productivity growth if the price of the input is identical in all its uses. Our empirical application, although very limited for the time being shows that both the choice of the approach and that of index number formula do matter.

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